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STRATIFICATION METHOD IN THE THEORY OF THIN SHELLS

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It is shown that on the basis of certain simplifications induced in the physical and geometrical dependences, such a "stratification" of a shell can be achieved for which the fibers of each of two layers will be deformed just as thin rods whose axes agree with the lines of principal curvature of the shell middle surface. The approach to analyzing shells on the basis of the relationships to be obtained below is called the "stratification method".

The question of the possibility of representing the computational schemes of plates and shells in the form of a set of intersecting mutually orthogonal rods was, as is known [1], of interest to L. Euler and J. Bernoulli. In connection with the appearance of modern high-speed computers, this idea was discussed in detail in [2 — 5]. A method was proposed in [6] to decompose the defferential equations of shell theory whereupon the shell would be reduced to a four-layer quasi-rod system continuous in each layer. Another variation of the decomposition method is given in [7]. It should be noted that the diagonal-free rod analogy was substantially found considerably earlier [8] for flexible orthotropic plates.

Taking account of the Poisson's ratio represents the greatest difficulty in constructing rod models. It has been shown on the basis of the V. Z. Vlasov equations of technical shell theory [9] that the membrane stresses in a shell are independent of the Poisson's ratio upon giving all the tangential boundary conditions in forces. It hence follows that if the tangential boundary conditions are kinematic, then taking account of the elastic constants inexactly can affect the stresses noticeably only near the reference contour and does not affect the membrane state of stress of the shell as a whole. In this connection, it is possible to neglect the influence of the Poisson's ratio in the physical dependences between the normal forces and their corresponding strains. Taking correct account of the Poisson's ratio turns out to be completely realizable in considering the shears of the shell middle surface, as well as its bending and torsion.

As is usually done, we write the shell strain potential energy in the form of a sum $U = U_\varkappa + U_\varepsilon$, where, (all the notation is standard)

$$U_\varkappa = \frac{1}{2}\iint\limits_\Omega D\,[\varkappa_1^2 + 2\nu\varkappa_1\varkappa_2 + \varkappa_2^2 + 2(1-\nu)\chi^2]\,ds_1\,ds_2$$

$$U_\varepsilon = \frac{1}{2}\iint\limits_\Omega \left[\frac{Eh}{1-\nu^2}(\varepsilon_1^2 + 2\nu\varepsilon_1\varepsilon_2 + \varepsilon_2^2) + Gh\omega^2\right] ds_1\,ds_2$$

Without limiting the generality of the final results, we take $\alpha_1 = s_1$ and $\alpha_2 = s_2$ as the quantities characterizing the position of point of the middle surface in a curvilinear coordinate system, where s_i is the arclength measured along the coordinate line $\alpha_{3-i} = \text{const}$, from a certain curve lying entirely within the same surface; the Lamé parameters $A_1 = A_2 = 1 = \text{const}$ and the curvature and torsion strains in $U_\varkappa$ are the following:

$$\varkappa_1 = -\frac{\partial^2 w}{\partial s_1^2} + \frac{1}{R_1}\frac{\partial u}{\partial s_1} - \frac{u}{R_1^2}\frac{\partial R_1}{\partial s_1}$$
$$\varkappa_2 = -\frac{\partial^2 w}{\partial s_2^2} + \frac{1}{R_2}\frac{\partial v}{\partial s_2} - \frac{v}{R_2^2}\frac{\partial R_2}{\partial s_2}$$
$$\chi = -\frac{\partial^2 w}{\partial s_1 \partial s_2} + \frac{1}{R_1}\frac{\partial u}{\partial s_2} + \frac{1}{R_2}\frac{\partial v}{\partial s_1}$$

The second terms in the expressions for $\varkappa_1, \varkappa_2$ as well as the second and third terms in the expression for χ are on the order of ε_i / R_j (it is assumed finally that all kinds of strain are of identical order of smallness, respectively, i.e., for instance, $\partial u / \partial s_1 \sim \partial u / \partial s_2 \sim \partial v / \partial s_1$, etc). One of the results of [10] is that in the formulas for the bending strain parameters, it is allowable to discard components of the order of ε_i / R_j without reducing the order of the error determined by using the Kirchhoff — Love hypotheses. Taking the above into account, it can be assumed that

$$U_\varkappa = \frac{1}{2}\iint_\Omega \left[D\left(\frac{\partial^2 w}{\partial s_1^2} + \frac{R_{11}}{R_1^2}u\right)^2 + 2\nu\left(\frac{\partial^2 w}{\partial s_1^2} + \frac{R_{11}}{R_1^2}u\right)\left(\frac{\partial^2 w}{\partial s_2^2} + \frac{R_{22}}{R_2^2}v\right) + \right.$$
$$\left.\left(\frac{\partial^2 w}{\partial s_2^2} + \frac{R_{22}}{R_2^2}v\right)^2 + 2(1-\nu)\left(\frac{\partial^2 w}{\partial s_1 \partial s_2}\right)^2\right] ds_1\, ds_2 \quad \left(R_{ij} = \frac{\partial R_i}{\partial s_j}\right)$$

In addition to $U_\varkappa$ we consider a somewhat different functional

$$U_\varkappa{}^* = \frac{1}{2}\iint_\Omega D\left[\left(\frac{\partial^2 w}{\partial s_1^2}\right)^2 + 2\left(\frac{\partial^2 w}{\partial s_1 \partial s_2}\right)^2 + \left(\frac{\partial^2 w}{\partial s_2^2}\right)^2 + \right.$$
$$2\left(\frac{R_{11}}{R_1^2}\frac{\partial^2 w}{\partial s_1^2}u + \frac{R_{22}}{R_2^2}\frac{\partial^2 w}{\partial s_2^2}v\right) + \left(\frac{R_{11}}{R_1^2}\right)^2 u^2 + \left(\frac{R_{22}}{R_2^2}\right)^2 v^2 +$$
$$\left. 2\nu\left(\frac{R_{22}}{R_2^2}\frac{\partial^2 w}{\partial s_1^2}v + \frac{R_{11}}{R_1^2}\frac{\partial^2 w}{\partial s_2^2}u + \frac{R_{11}R_{22}}{R_1^2 R_2^2}uv\right)\right] ds_1\, ds_2$$

It is easy to see by direct substitution that all the Euler equations for $U_\varkappa{}^*$ (for a variation in the kinematic parameters) agree completely with the corresponding equations for $U_\varkappa$. Hence, keeping in mind obtaining the equation in displacements by a variational means, we shall use a new, more convenient functional for this investigation in writing the strain potential energy (here the absence of the product $(\partial^2 w / \partial s_1^2) \times (\partial^2 w / \partial s_2^2)$) in $U_\varkappa{}^*$ turns out to be essential).

Taking account of the geometric dependences for the angles of rotation of the normals ϑ_1 and ϑ_2, and discarding terms of the order of ε_i / R_j in the expressions relating these angles to the mixed derivative $\partial^2 w / \partial s_1 \partial s_2$, we can assume in the functional $U_\varkappa{}^*$

$$2\left(\frac{\partial^2 w}{\partial s_1 \partial s_2}\right)^2 = \left(\frac{\partial \vartheta_1}{\partial s_2} + \frac{R_{12}}{R_1^2}u\right)^2 + \left(\frac{\partial \vartheta_2}{\partial s_1} + \frac{R_{21}}{R_2^2}v\right)^2$$

by appending the equalities

$$\vartheta_1 = -\frac{\partial w}{\partial s_1} + \frac{u}{R_1}, \quad \vartheta_2 = -\frac{\partial w}{\partial s_2} + \frac{v}{R_2}$$

as additional conditions.

Turning to the functional U_ε, we can, in conformity with the simplification stipulated above, neglect the influence of the Poisson's ratio, i.e., discard the component $2\nu\varepsilon_1\varepsilon_2$. We write the term dependent on the shear strain as

$$Gh\omega^2 = 2Gh\left[\left(\frac{\partial v}{\partial s_1} - \vartheta_n\right)^2 + \left(\frac{\partial u}{\partial s_2} + \vartheta_n\right)^2\right]$$

where the angle of rotation of a shell element around the normal is expressed by the formula [10]

$$\vartheta_n = \frac{1}{2}\left(\frac{\partial v}{\partial s_1} - \frac{\partial u}{\partial s_2}\right)$$

For the usual expressions for ε_1, ε_2 we have

$$U_\varepsilon = \frac{1}{2}\iint\limits_\Omega \Big\{E^*h\left[\left(\frac{\partial u}{\partial s_1} + \frac{w}{R_1}\right)^2 + \left(\frac{\partial v}{\partial s_2} + \frac{w}{R_2}\right)^2\right] +$$

$$2Gh\left[\left(\frac{\partial v}{\partial s_1} - \vartheta_n\right)^2 + \left(\frac{\partial u}{\partial s_2} + \vartheta_n\right)^2\right]\Big\} ds_1\, ds_2, \quad E^* = \frac{E}{1-\nu^2}$$

If the distributed loads q_1, q_2 and q_n as well as the distributed moments m_1, m_2 act on the shell, then the potential (total energy) of the system is

$$\Pi = U - \iint\limits_\Omega [q_1 u + q_2 v + q_n w + m_1\vartheta_1 + m_2\vartheta_2]\, ds_1\, ds_2$$

where it can be considered that contour forces and moments applied to the free edges are included in the work of the external loads by using the delta function.

Let us represent Π as the sum of two components

$$\Pi = \Pi_1 + \Pi_2, \quad \Pi_i = U_{\varkappa i}{}^* + U_{\varepsilon i} + \Delta\Pi_i$$

We write down in detail only the expressions for the terms forming the first component

$$U_{\varkappa 1}^* = \frac{1}{2}\iint\limits_\Omega D\left[\left(\frac{\partial^2 w^{(1)}}{\partial s_1^2}\right)^2 + \left(\frac{\partial\vartheta_2^{(1)}}{\partial s_1} + \frac{R_{21}}{R_2^2}v^{(1)}\right)^2 + \right.$$

$$2\frac{R_{11}\partial^2 w^{(1)}}{R_1^2\,\partial s_1^2}u^{(1)} + \left(\frac{R_{11}}{R_1^2}u^{(1)}\right)^2 +$$

$$\left. 2\nu\left(\frac{R_{22}}{R_2^2}\frac{\partial^2 w}{\partial s_1^2}v^{(1)} + \frac{1}{2}\frac{R_{11}R_{22}}{R_1^2R_2^2}u^{(1)}v^{(1)}\right)\right] ds_1\, ds_2$$

$$U_{\varepsilon 1} = \frac{1}{2}\iint\limits_\Omega \left[E^*h\left(\frac{\partial u^{(1)}}{\partial s_1} + \frac{w^{(1)}}{R_1}\right)^2 + 2Gh\left(\frac{\partial v^{(1)}}{\partial s_1} - \vartheta_n^{(1)}\right)^2\right] ds_1\, ds_2$$

$$\Delta\Pi_1 = -\iint\limits_\Omega \left[q_1 u^{(1)} + m_2\vartheta_2^{(1)} + \frac{1}{2}q_n w^{(1)}\right] ds_1\, ds_2$$

Here the superscript in the notations of the kinematic parameters indicates the number of the layer (1 or 2) to which this parameter belongs.

The work of the loads q_1 and m_2 is referred to the first layer and the work of the loads q_2 and m_1 to the second upon insertion of the functionals $\Delta\Pi_1$ and analogously $\Delta\Pi_2$. The work of the loads q_n is divided equally between the two layers. Since the similar parameters of both layers should be equal, then the left sides of the appropriate additional conditions with the Lagrangean multipliers λ_1, λ_2, λ_3, μ_1, μ_2, μ_3 are included in the final functional Φ to be minimized

$$\Phi = \Pi + \iint_\Omega \Big\{\lambda_1 (u^{(1)} - u^{(2)}) + \lambda_2 (v^{(1)} - v^{(2)}) + \lambda_3 (w^{(1)} - w^{(2)}) + \mu_1 \left[\vartheta_1^{(2)} + \frac{\partial w^{(1)}}{\partial s_1} - \frac{u^{(1)}}{R_1}\right] + \mu_2 \left[\vartheta_2^{(1)} + \frac{\partial w^{(2)}}{\partial s_2^2} - \frac{v^{(2)}}{R_2}\right] + \mu_3 (\vartheta_n^{(1)} - \vartheta_n^{(2)})\Big\} ds_1 ds_2$$

Varying Φ over all the kinematic parameters and the Lagrangean multipliers, we obtain a complete system of equations to determine these unknowns. Let us just indicate the result of the variation over the kinematic parameters referring to the first layer (i.e., over $w^{(1)}$, $\vartheta_2^{(1)}$, $u^{(1)}$, $v^{(1)}$, $\vartheta_n^{(1)}$)

$$\frac{\partial^2}{\partial s_1^2}\left[D\left(\frac{\partial^2 w^{(1)}}{\partial s_1^2} + \frac{R_{11}}{R_1^2} u^{(1)} + \nu \frac{R_{22}}{R_2^2} v^{(1)}\right)\right] + \frac{E^*h}{R_1}\left(\frac{\partial u^{(1)}}{\partial s_1} + \frac{w^{(1)}}{R_1}\right) = \frac{1}{2} q_n - \lambda_3 + \frac{\partial \mu_1}{\partial s_1} \tag{1}$$

$$-\frac{\partial}{\partial s_1}\left[D\left(\frac{\partial \vartheta_2^{(1)}}{\partial s_1} + \frac{R_{21}}{R_2^2} v^{(1)}\right)\right] = m_2 - \mu_2$$

$$-\frac{\partial}{\partial s_1}\left[E^*h\left(\frac{\partial u^{(1)}}{\partial s_1} + \frac{w^{(1)}}{R_1}\right)\right] + D\left[\frac{R_{11}}{R_1^2}\frac{\partial^2 w}{\partial s_1^2} + \left(\frac{R_{11}}{R_1^2}\right)^2 u^{(1)} + \frac{\nu}{2}\frac{R_{11}R_{22}}{R_1^2 R_2^2} v^{(1)}\right] = q_1 - \lambda_1 + \frac{\mu_1}{R_1}$$

$$-\frac{\partial}{\partial s_1}\left[2Gh\left(\frac{\partial v^{(1)}}{\partial s_1} - \vartheta_n^{(1)}\right)\right] + D\left[\frac{R_{21}}{R_2^2}\left(\frac{\partial \vartheta_2^{(1)}}{\partial s_1} + \frac{R_{21}}{R_2^2} v^{(1)}\right) + \nu \frac{R_{22}}{R_2^2}\frac{\partial^2 w^{(1)}}{\partial s_1^2} + \frac{\nu}{2}\frac{R_{11}R_{22}}{R_1^2 R_2^2} u^{(1)}\right] = q_2 - \lambda_2$$

$$2Gh\left(\frac{\partial v^{(1)}}{\partial s_1} - \vartheta_n^{(1)}\right) = -\mu_3$$

The first of these equations describes the bending of infinitely narrow "rods" of the first layer, the second describes their **torsion**, the third the tension, the fourth and fifth the shear strain in the shell middle surface.

Let us clarify the physical meaning of the Lagrangean multiplier μ_1. It is known that the torque is replaced by equivalent shear and transverse forces in formulating the boundary conditions. Completely analogously, the exterior (relative to the rods of the first layer) linear moment μ_1 can be replaced by the normal load $q_n^\circ = \partial\mu_1 / \partial s_1$ and the tangential load $q_1^\circ = \mu_1 / R_1$ (Fig. 1). The loads mentioned are in the first and third equations of (1).

Thus, the moment μ_1 acts on the "rods" of the first direction in the normal plane containing the direction $\mathbf{e}_1$. It turns out that in the variation with respect to $\vartheta_1^{(2)}$ a moment $(-\mu_1)$ acts at the same point on the rod of the second direction in the same plane. In other words μ_1 is the moment of the interaction between rods of two layers which assures the equality of the angles of rotation of the normal around the direction $\mathbf{e}_2$. The moment μ_2 performs an analogous role; λ_1, λ_2, λ_3 are the tangential and normal interaction loads, and μ_3 is the moment acting in a plane tangent to the middle surface.

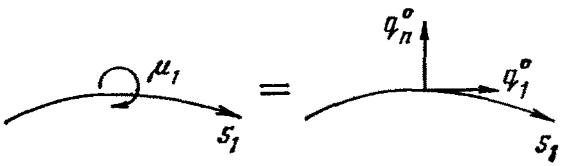

Fig. 1

In order to clarify the singularities of the work of the "stratified" shell in shear, we consider a rod whose axis agrees with the contour of the cylindrical shell cross-section (i.e., we set $R_2 = \infty$) and we consider the shell thickness h constant. The fourth equation then becomes

$$-2Gh\left(\frac{\partial^2 v^{(1)}}{\partial s_1^2} - \frac{\partial \vartheta_n^{(1)}}{\partial s_1}\right) = q_2 - \lambda_2$$

or

$$-Gh\frac{\partial \omega}{\partial s_1} = q_2 - \lambda_2$$

where the shear strain ω should be identical for rods of both directions upon compliance with the additional conditions included in Φ. This last equation shows that the shear forces $S_{12} = Gh\omega$ (Fig. 2) are equilibrated by the given load q_2 and the interaction load λ_2. The fifth equation can be represented in the form:

$$S_{12} + \mu_3 = 0$$

from which it is seen that the interaction moment μ_3 cancels the rotating effect of the shear forces. Therefore, the moment μ_3 replaces action of the shear forces

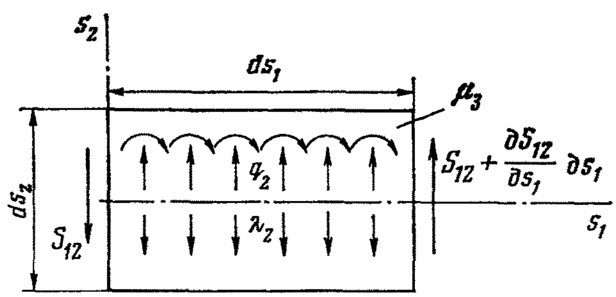

Fig. 2

S_{21} paired with S_{12} for the first layer since the former are absent in rods of this layer.

As was mentioned, since the contour forces were included in the functional $\Delta\Pi_i$, then compliance with the static boundary conditions in the Saint Venant sense is assured: at those edges where the loads are given and the displacements are varied, the internal forces in the rods are statically equivalent to these loads. The kinematic boundary conditions are easily taken into account by giving the rods the appropriate reference framing.

Therefore, there are two families of rods, between which six ordinary (rod) interaction forces act at each point of the middle surface. For comparison, we mention that the number of interaction forces is thirteen in the method of separation [6], and nine for the zero Poisson's ratio in all the physical dependences.

Let us present three simple examples of stratification of two-dimensional objects.

1°. We set $s_1 \equiv x,\ s_2 \equiv y,\ m_1 = m_2 = u = v = \vartheta_n = 0$ under the action of a normal load q_n on a plate. The equations to which the rods of each of the layers are subject have the form

$$D\frac{\partial^4 w^{(1)}}{\partial x^4} = \frac{1}{2}q_n - \lambda_3 + \frac{\partial \mu_1}{\partial x}, \quad D\frac{\partial^2 \vartheta_2^{(1)}}{\partial x^2} = \mu_2$$

$$D\frac{\partial^4 w^{(2)}}{\partial y^4} = \frac{1}{2}q_n + \lambda_3 + \frac{\partial \mu_2}{\partial y}, \quad D\frac{\partial^2 \vartheta_1^{(2)}}{\partial y^2} = \mu_1$$

under the additional conditions

$$w^{(1)} = w^{(2)}, \quad \vartheta_2^{(1)} = -\frac{\partial w^{(2)}}{\partial y}, \quad \vartheta_1^{(2)} = -\frac{\partial w^{(1)}}{\partial x}$$

These equations describe the bending and torsion of a plane orthogonal system of straight beams and are, as a set, equivalent to the Sophie Germain equation.

2°. For the axisymmetric strain of a circular cylindrical shell of radius R with axis parallel to the z axis of a Cartesian coordinate system, we obtain under the action of normal and longitudinal loads

$$D\frac{d^4 w^{(1)}}{dz^4} = \frac{1}{2}q_n - \lambda_3, \quad -E^*h\frac{d^2 u^{(1)}}{dz^2} = q_1$$

$$\frac{E^*h}{R^2}w^{(2)} = \frac{1}{2}q_n + \lambda_3$$

The first two equations describe the bending, and tension of the longitudinal rods, respectively, and the third is for the compression of the circumferential rods. It is clear that the set of the first and third equations (under the condition $w^{(1)} = w^{(2)}$) is equivalent to the usual equation of symmetric cylindrical shell bending.

3°. For a shallow shell under the load q_n we take $s_1 \equiv x_1, s_2 \equiv x_2, u \equiv u_1, v \equiv u_2$. Let us also assume that $R_{ij} \approx 0, \mu_i / R_i \approx 0 \quad \forall i, j$. Then a system of equations

$$D\frac{\partial^4 w^{(i)}}{\partial x_i^4} + \frac{E^*h}{R_i}\varepsilon_i - \frac{1}{2}q_n - (-1)^i\lambda_3 - \frac{\partial \mu_i}{\partial x_i} = 0 \tag{2}$$

$$-D\frac{\partial^2 \vartheta_{3-i}^{(i)}}{\partial x_i^2} + \mu_{3-i} = 0, \quad -E^*h\frac{\partial \varepsilon_i}{\partial x_i} + (-1)^i\lambda_i = 0$$

$$2Gh\frac{\partial\omega^{(i)}}{\partial x_i} + (-1)^i \lambda_{3-i} = 0, \quad -Gh\omega^{(i)} + \mu_3 = 0$$

is obtained for the rods of each of the directions $(i = 1, 2)$.

From the last relationship follows $\omega^{(1)} = \omega^{(2)} \equiv \omega$, and furthermore $S_{12} = S_{21} = Gh\omega \equiv S$. Taking $N_1 = E^*h\varepsilon_1$ and $N_2 = E^*h\varepsilon_2$, we obtain the possibility of introducing the stress function Ψ. The tangential equilibrium equations are thereby satisfied automatically, which can be considered the result of the third and fourth relationships in (2) in this case. The appropriate combination of the first and second equations in (2) yield (for $w^{(1)} = w^{(2)} \equiv w$, $\vartheta^{(i)}_{3-i} = -\partial w / \partial x_{3-i}$)

$$D\nabla^4 w - \frac{1}{R_1}\frac{\partial^2\Psi}{\partial x_2^2} - \frac{1}{R_2}\frac{\partial^2\Psi}{\partial x_1^2} = q_n$$

To obtain the second governing equation of shallow shell theory, we take into account that from the geometric dependences

$$\varepsilon_i = \frac{\partial u^{(i)}}{\partial x_i} + \frac{w^{(i)}}{R_i}, \quad \omega^{(i)} = 2\left[\frac{\partial u^{(i)}_{3-i}}{\partial x_i} + (-1)^i \vartheta_n^{(i)}\right]$$

the strain compatibility condition

$$\frac{\partial^2\varepsilon_1}{\partial x_2^2} + \frac{\partial^2\varepsilon_2}{\partial x_1^2} - \frac{\partial^2\omega}{\partial x_1\partial x_2} = \frac{1}{R_1}\frac{\partial^2 w}{\partial x_2^2} + \frac{1}{R_2}\frac{\partial^2 w}{\partial x_1^2}$$

follows under the additional requirements $u_i^{(1)} = u_i^{(2)}$, $\vartheta_n^{(1)} = \vartheta_n^{(2)}$, from which the desired relationship is later indeed obtained by the usual means. The set of systems of the type (2) for rods of two directions is therefore equivalent to the system of governing equations of shallow shell theory.

In conclusion, let us note that effective algorithms for the approximate analysis of shells as conditional rod systems can be constructed on the basis of the mentioned method of stratification.

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